# What do we mean by 1=0.9 reoccurring

## A conversation with a none Mathematician  who will not believe it

# How I first came across this idea

- At Open University Summer School for the Maths foundation Course
- Every Morning 15 minutes of Quick fire questions such as
  - Is 1 > 0.9 reoccurring
- Answer 1= 0.9 recurring
- Proof they gave
  - If two real numbers are different then by the density property of the real numbers there is a third number which is the average of the two sitting between them
  - But 1 + 0.9… = 1.9… and 1.9../2 = .9… hence 1 = 0.9..

# Reaction of my friend

- Very intelligent
- First in History and politics
- But claimed he got a head ache when he considered or thought of any mathematical ideas
- His reaction
  - Disbelief
  - This is some sort of mathematical slight of hand
  - No  way they can be the same
- My reaction :- come up with further proofs to try and convince him

# Two further proofs

- Proof 1
  - Let s = 0.9.. = 9/10 + 9/100 +9/1000 + …
  - 10s = 9 + 9/10 + 9/100 + 9/1000+ …
  - 10s = 9 + s => 9s = 9 => s = 1
- Proof 2
  - 1/3 = 0.3..
  - 3X1/3 = 3 x 0.3. .
  - 1 = 0.9..
- My friends reaction
  - Just further Mathematical slight of hand

# So why does my Friend not accept these proofs ?

- All three proofs assume
  - that numbers like 0.9.. and 0.3.. Exist
  - That you can treat then algebraically in the same way as none-reoccurring numbers
- Are these assumptions valid or was my friend right to be cynical
- What happens if we apply a similar technique the other way that is to .9.0
  - let s =. .9.0 = 9 +9X10 + 9X100 + 9 X1000 ….
  - 10s = 9X10 + 9 X100 +9X1000.. = s -9
  - 9s = -9 => s = -1
  - Clearly not a reasonable answer !!
- What about ...9.9… apply the same reasoning we get S = 0
- So it seems we are on dangerous ground when we try to apply algebraic procedure to reoccurring numbers
- Seems my friend was right

# What is the difference. Introducing limits

- For …9.0  s does not exist
  - We have the sequence  , 9, 90, 900, 9000
  - Each term in this sequence is larger then the pervious term so by the divergence test the series  formed by the partial sums of his sequence  is divergent so has no limit, so in this case  s does not exist
- For 0.9..  s may exist
  - We have the sequence 9/10, 9/100, 9/1000,9/10,000
  - Each term in the sequence is small than the previous term so in this case the divergence test is inconclusive and the series formed by partial sums of this sequence may converge hence s may exist

# Does s exist for 0.9.. Have a limit and if so what is it ?

- Let the sequence Xn consisting  9/10,9/100, 9/1000……
- X(n+1)/Xn  = 1/10 for all n
- So by the Ratio test The series Sum(Xn) for n=1 to infinity is absolutely convergence so  has a limit.
- To show that this limit is 1 we can use limit of sum(an) n= 0 to infinity = a/(1-r) with  a = 9 ,r = 1/10
- $\sum_{n=0}^{\infty} {}^{9}/_{10^n}$ = **9 +** $\sum_{n=1}^{\infty} {}^{9}/_{10^n}$
- 9/(1-1/10) = 10 = **9 +** $\sum_{n=1}^{\infty} {}^{9}/_{10^n}$
- So $\sum_{n=1}^{\infty} {}^{9}/_{10^n}$ = 1

# So does it all mean

- 1= 0.9… are  we really saying the limit of the partial sums of the  sequence 9/10, 9/100 .. Is 1 and as this limit exist we can use algebraic methods to manipulate it
- Also that the limit of …99999.0 does not exist so we cannot use algebraic methods to manipulate it
- And For numbers with infinite decimal expansions we can only manipulate them using algebraic methods if they can be expressed as a convergent sequence with a limit

# So what does my friend say

- I was still unable to satisfy him
- But the approach using limits has been more satisfying for me
- I have also learnt the following
  - Talking with someone who sees mathematics as some sort of occult art and is cynical about some of its reasoning can be useful and lead to deeper insights
  - Even if a proof seems very convincing there may be hidden assumptions that need to made explicit
  - Methods that work for one type of number, decimal numbers with finite expansions may not work for others , decimal numbers with infinite expansions

# Further considerations

- Reoccurring expansions of fractions are a product of the number system being used.
  - Using a base 12 1/3 becomes 0.4, 2/3 = 0.8 so 3/3 = 1
  - But other fractions become recurring sequences
- What about other number systems
- Just come across Surreal numbers  and in this number system it seems that 1 and 0.99… are not equal ?
- In this system each real number is surrounded by surreal numbers which are closer to it than any other real numbers  0.99.. Can be expressed as a surreal number that is not equal to the surreal number 1