

Polyomino Collinearity

(or: The Joy of Recreational Maths)

Dots on a Tiling

2024 Competition

by

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Dots on a Tiling

Place points on the vertices of the tiling such that:


- 1) All points are connected by the edges of length 1.
- 2) No 4 points are colinear

How many can points can you place?

Prize: 3 mini Toblerones


Examples:

Wrong




Not connected

Wrong



4 colinear points

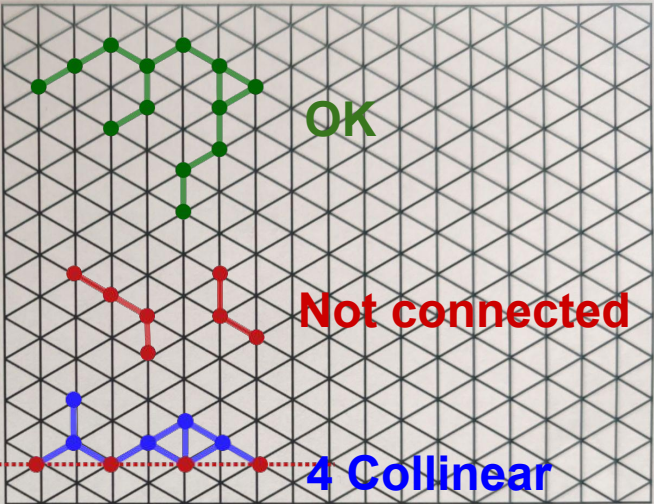
Right

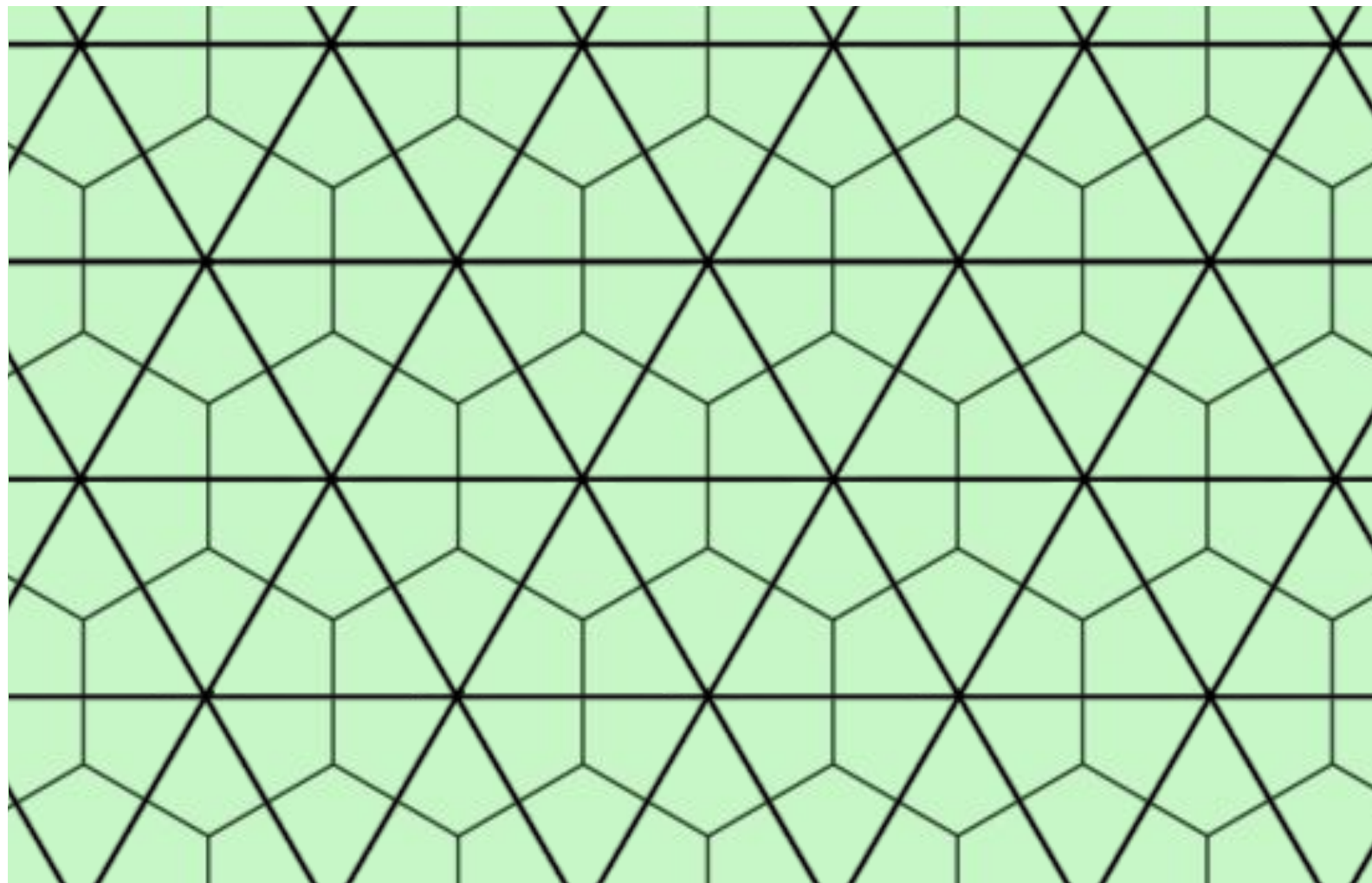


Right

Name _____

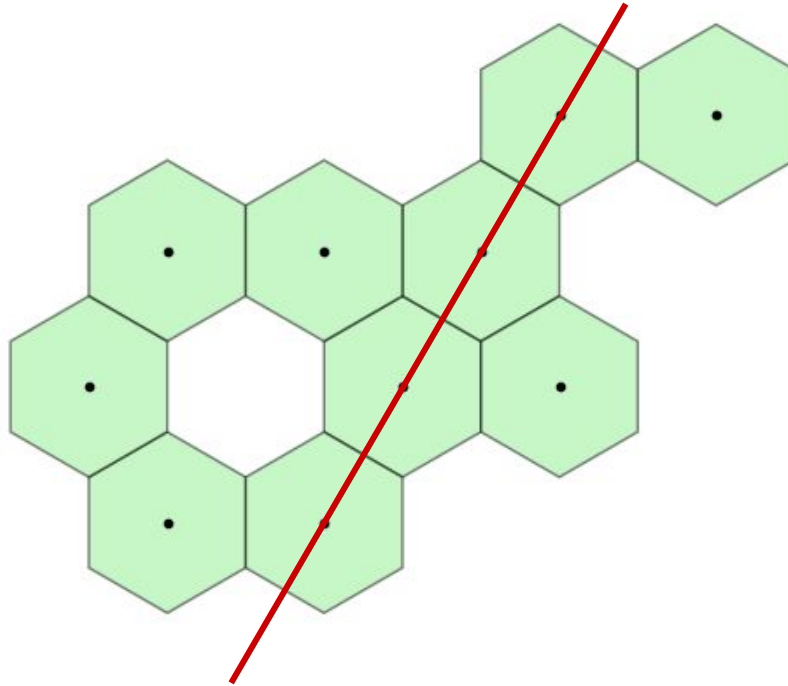
Points _____





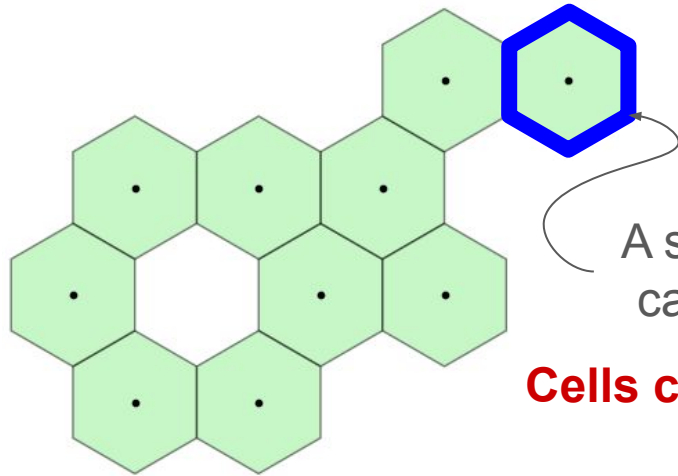
Make a single pattern of tiled hexagons

With no more than 3 tile centres on any line in the plane



Pattern of Tiles = Polyomino

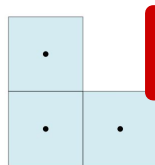
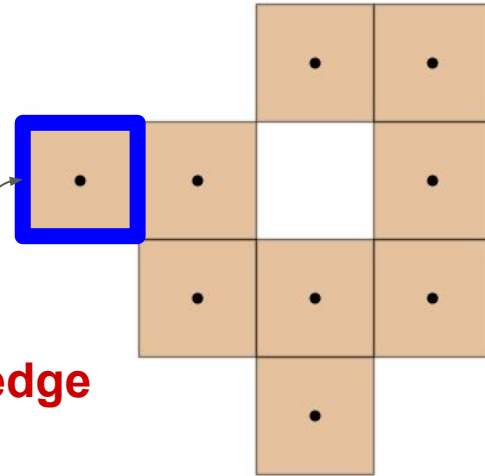
Hexagonal Polyomino (or Polyhex)



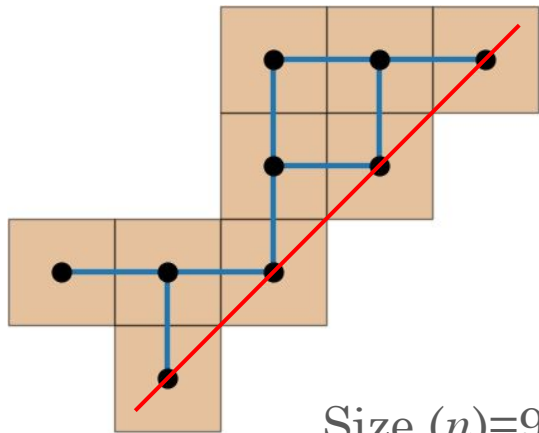
A single tile is called a **cell**

Cells connect via an edge

Square Polyomino (or Polyomino)



Not a polyomino - cell not connected



Size $(n)=9$

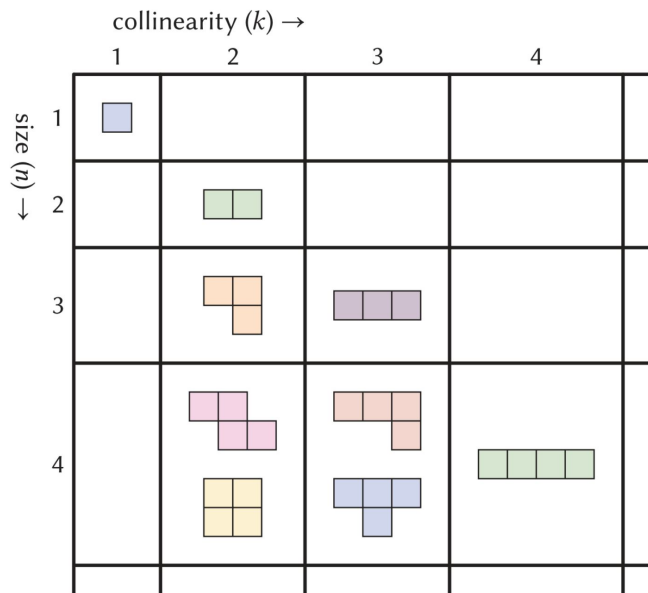
Collinearity (k)

The largest number of cell centres on any line in the plane.

$$k = 4$$

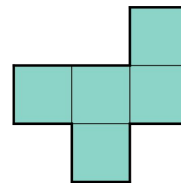
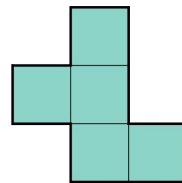
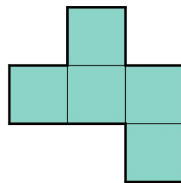
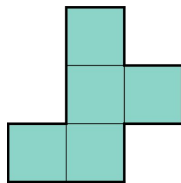
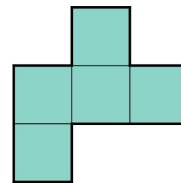
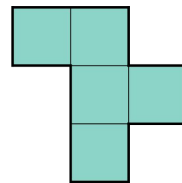
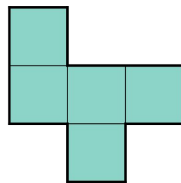
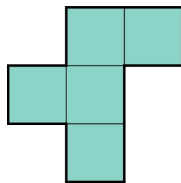
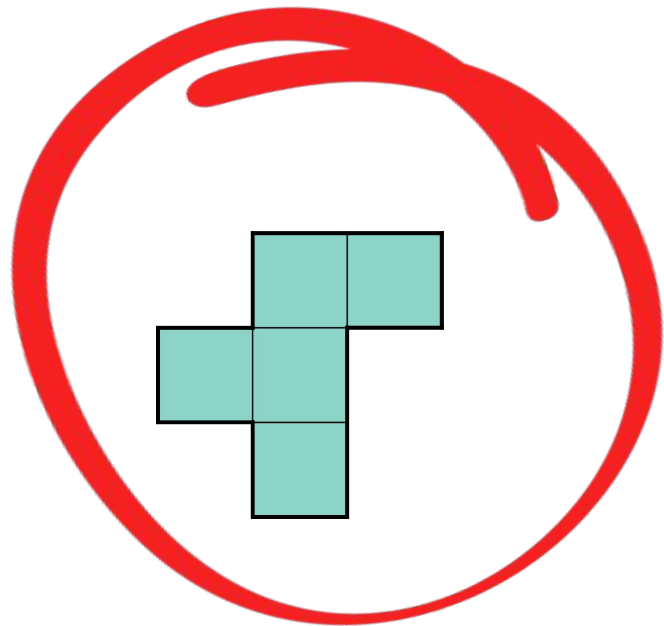
		collinearity $(k) \rightarrow$			
		1	2	3	4
size $(n) \downarrow$	1				
	2				
	3				
	4		 	 	

$$P(n,k) = \{ \text{polyominoes with } \mathbf{Size} = n \text{ and } \mathbf{Collinearity} = k \}$$

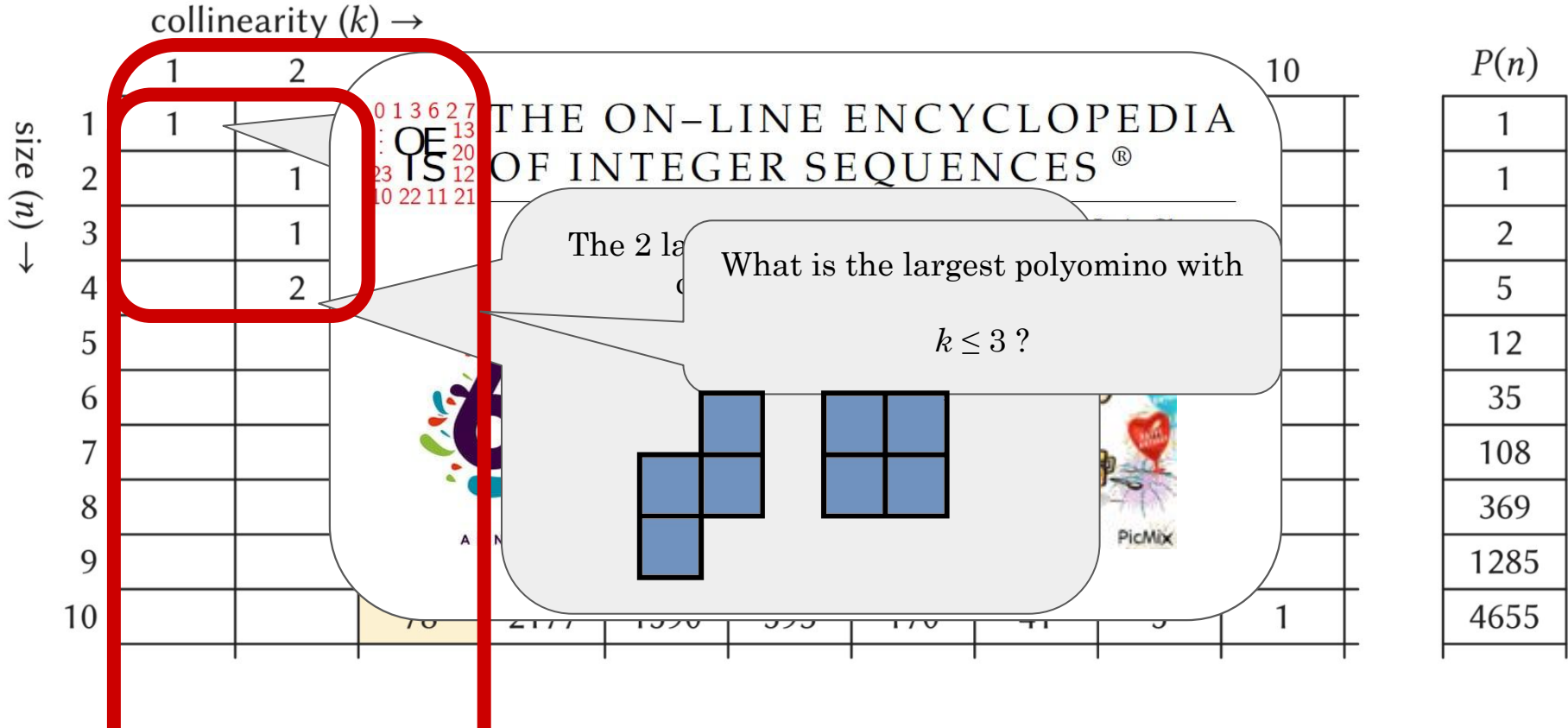


	k				
n	1	2	3	4	$ P(n) $
1	1				1
2		1			1
3		1	1		2
4		2	2	1	5

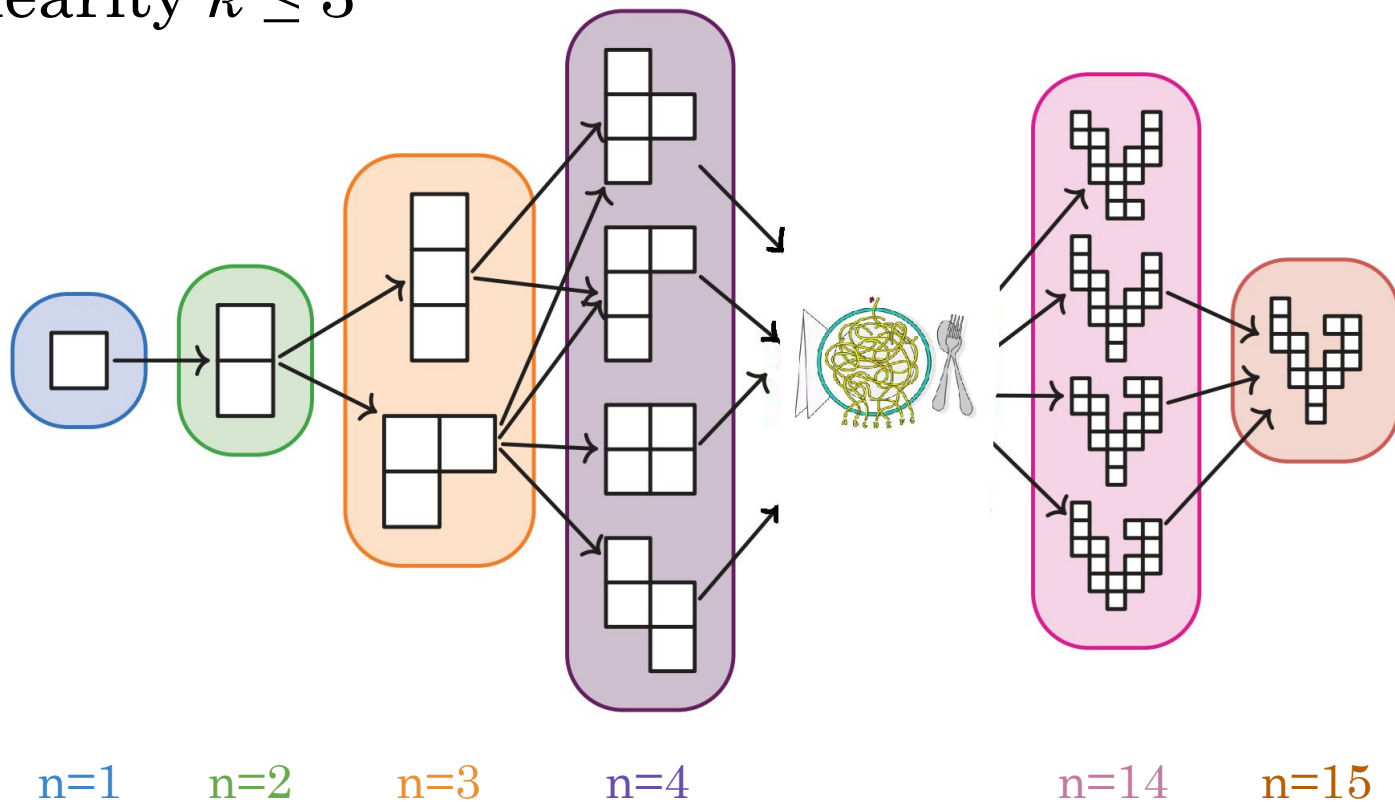
What am I Counting ?



$|P(n,k)|$ for the Square (oeis: A378169)

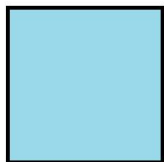


Collinearity $k \leq 3$



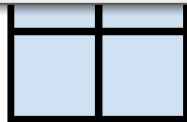
Largest square solutions for a limited collinearity (k)

$k \leq 1$



$n = 1$

$k \leq 2$

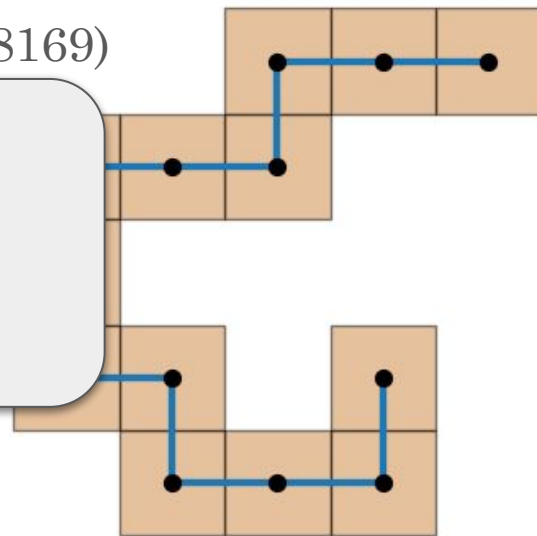


$n = 4$

$k \leq 3$

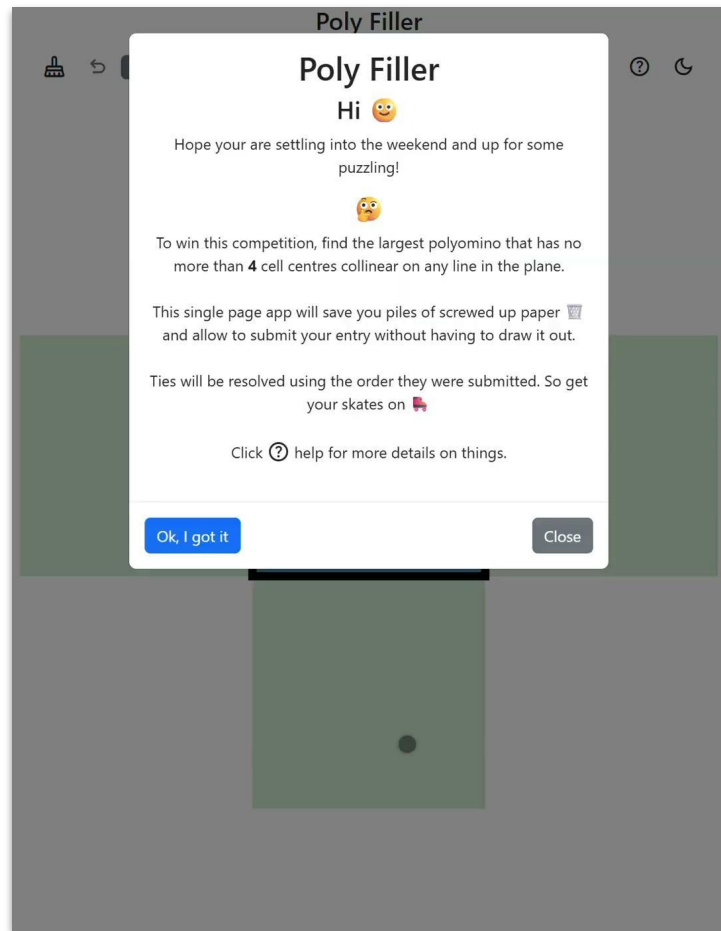
(oeis: A378169)

$k \leq 4 ?$



$n = 15$

tinyurl.com/comp-polyfiller



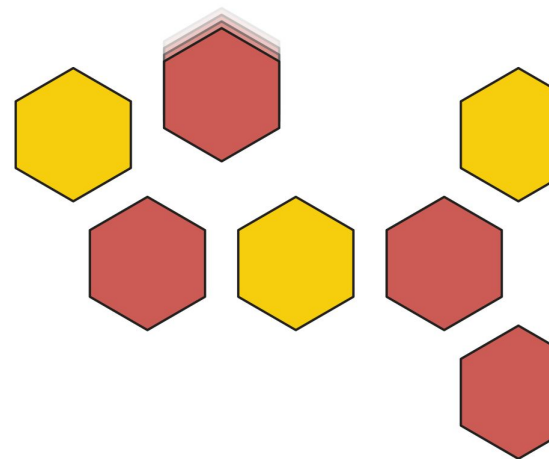


ISSUE 22



Don't connect four

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Hexagon: Largest solutions for $k \leq 3$

!! SPOILER ALERT !!

Dots on a Tiling

Place points on the vertices of the tiling such that:

- 1) All points are connected by the edges of length 1.
- 2) No 4 points are collinear.

Wrong

Not connected

Wrong

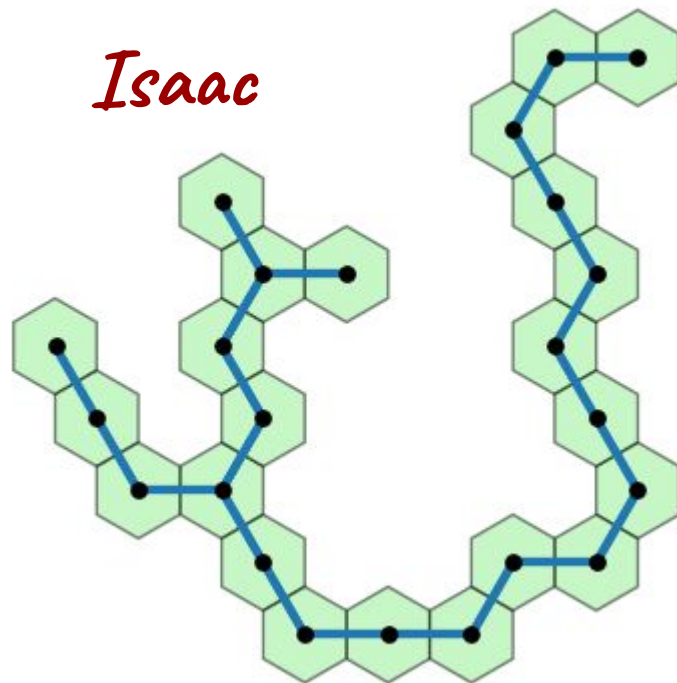
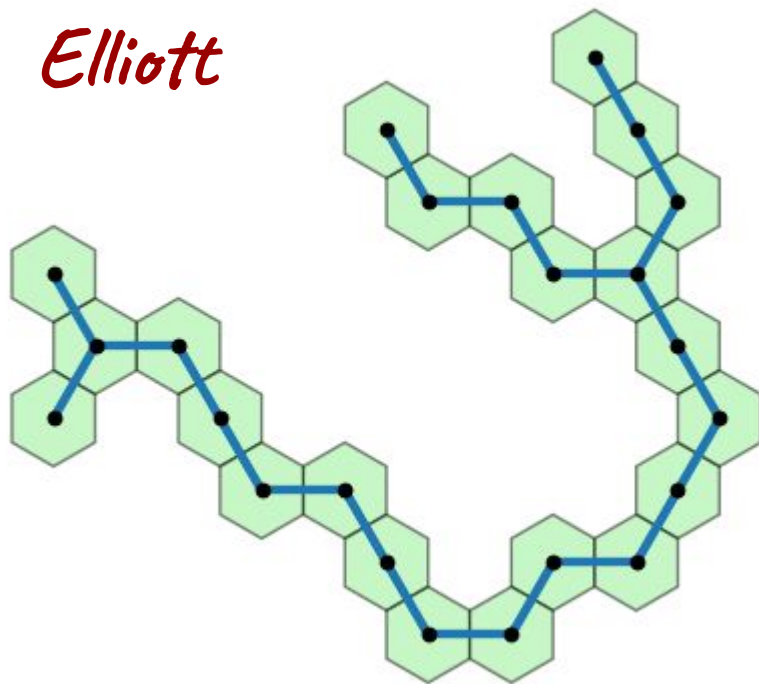
4 collinear points

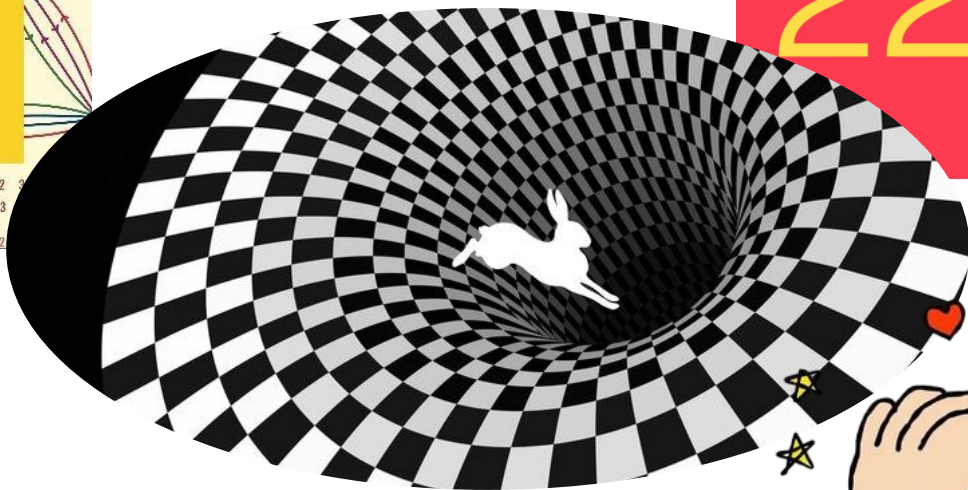
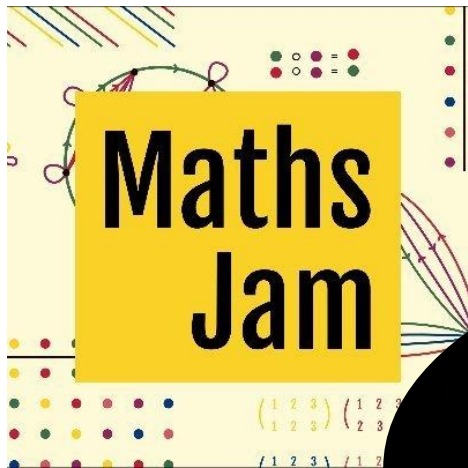
Right

Name _____

Points _____

Hexagon: Largest solutions for $k \leq 3$





$$\begin{matrix} 0 & 1 & 3 & 6 & 2 & 7 \\ : & & & & & 13 \\ : & \text{OEIS} & & & & 20 \\ 23 & & & & & 12 \\ 10 & 22 & 11 & 21 & & \end{matrix}$$

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