How large is a piece of paper?

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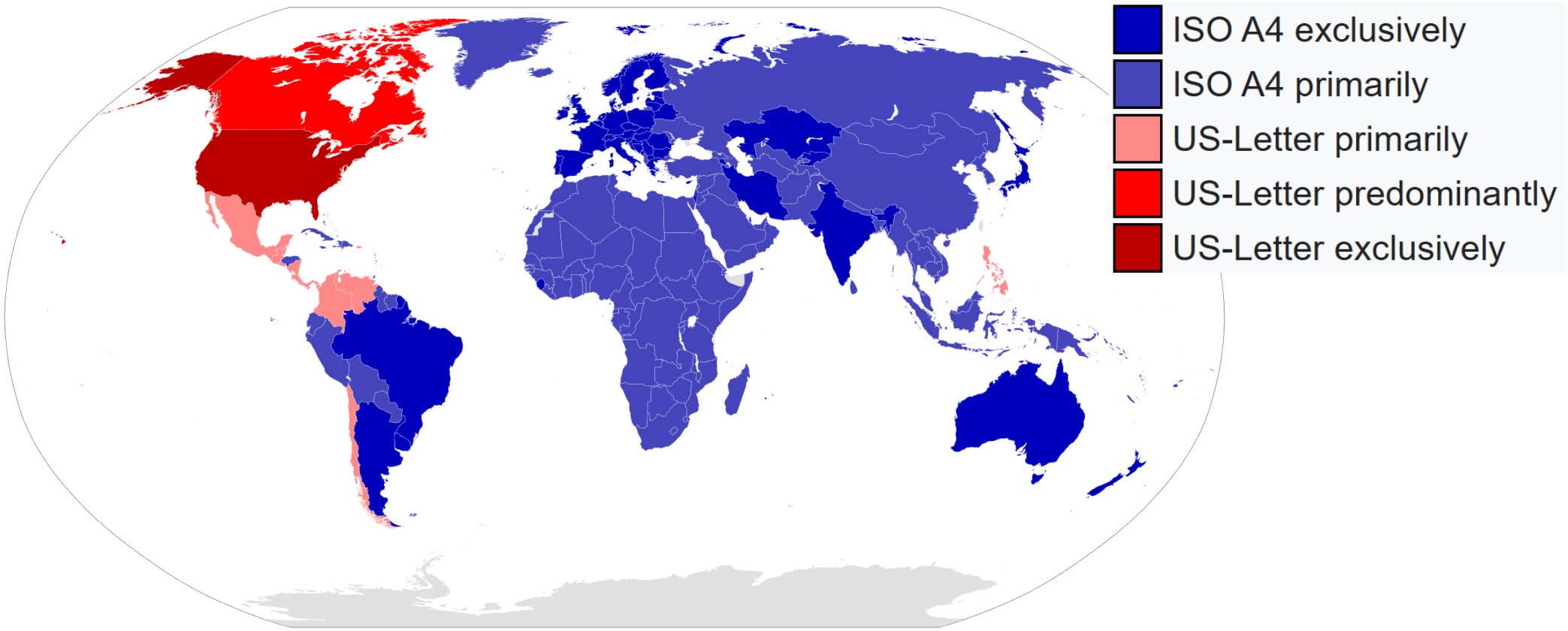
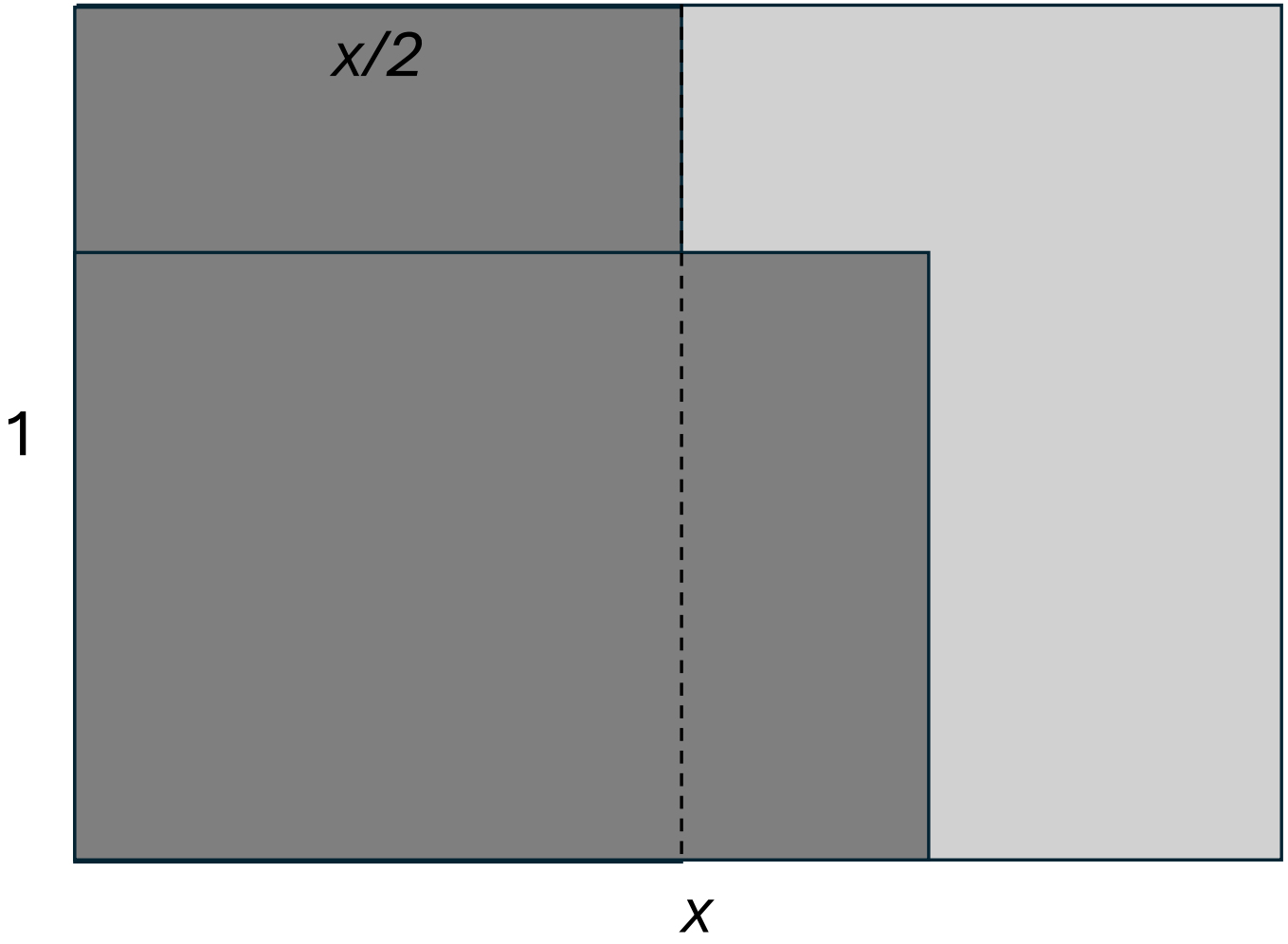
Most countries use ISO 216 (A series etc.)



Paper size - Wikipedia

ISO 216: Cutting in half maintains the aspect ratio

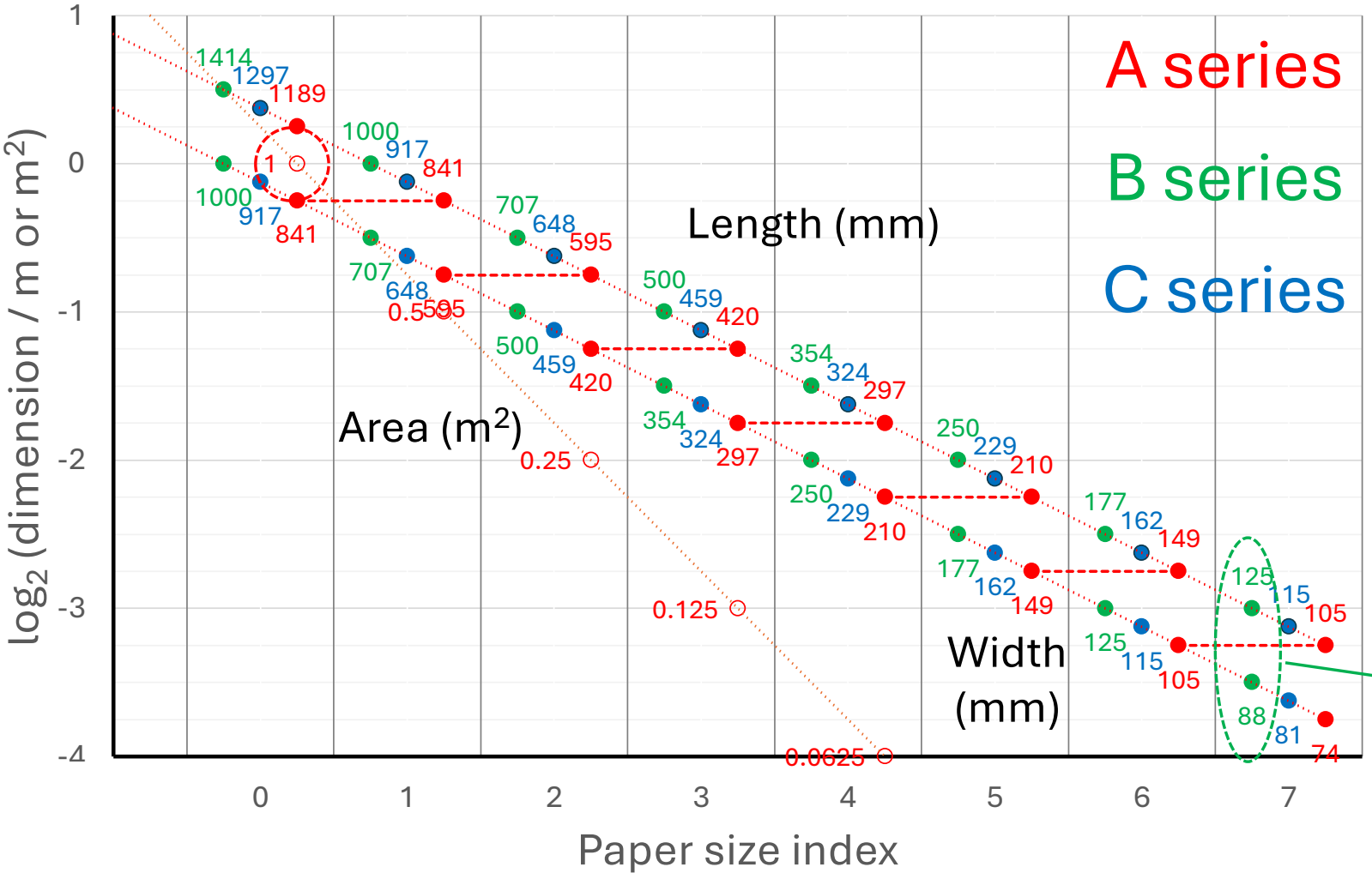


Aspect ratio x
$= 1/(x/2)$

$\therefore x = \sqrt{2}$

Ask me later about the history.

ISO 216 paper sizes



A0 has an area of 1 m^2

B sizes are the geometric mean of A sizes.

- *Effectively* $Bn \equiv An\text{-}½$

C sizes are the geometric mean of A and B sizes.

- $Cn \equiv An\text{-}¼$

Passport size B7 (≡A6.5)
ISO/IEC 7810 ID3

Size of An paper

$$\mathrm{Area/m^2} = 2^{-n}$$

$$\mathrm{Length/m} = 2^{-\frac{n}{2}+\frac{1}{4}}$$

$$\mathrm{Width/m} = 2^{-\frac{n}{2}-\frac{1}{4}}$$

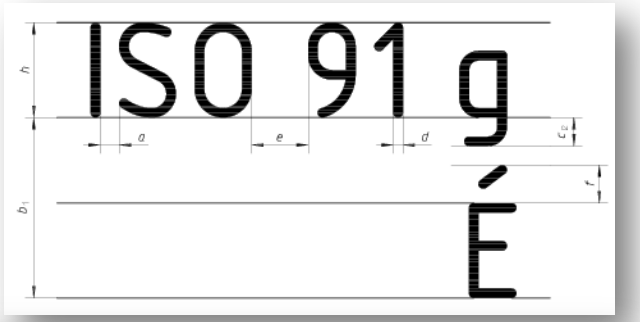
and Bn ≡ An-½, Cn ≡ An-¼

Technical drawing

- Technical pens (ISO 9175-1) and lettering (ISO 3098-0) also follow the $\sqrt{2}$ ratio.

Technical product documentation — Lettering —

Part 0:
General requirements



5.3 Range of nominal sizes

The range of nominal sizes is specified as follows:

1,8 mm; 2,5 mm; 3,5 mm; 5 mm; 7 mm; 10 mm; 14 mm; 20 mm

The multiple of $\sqrt{2}$ in the range of heights for lettering is derived from the standardized progression of dimensions for paper sizes (see ISO 216).

The line widths shall be in accordance with ISO 128-20 and the same line width shall be used for both upper-case and lower-case letters.

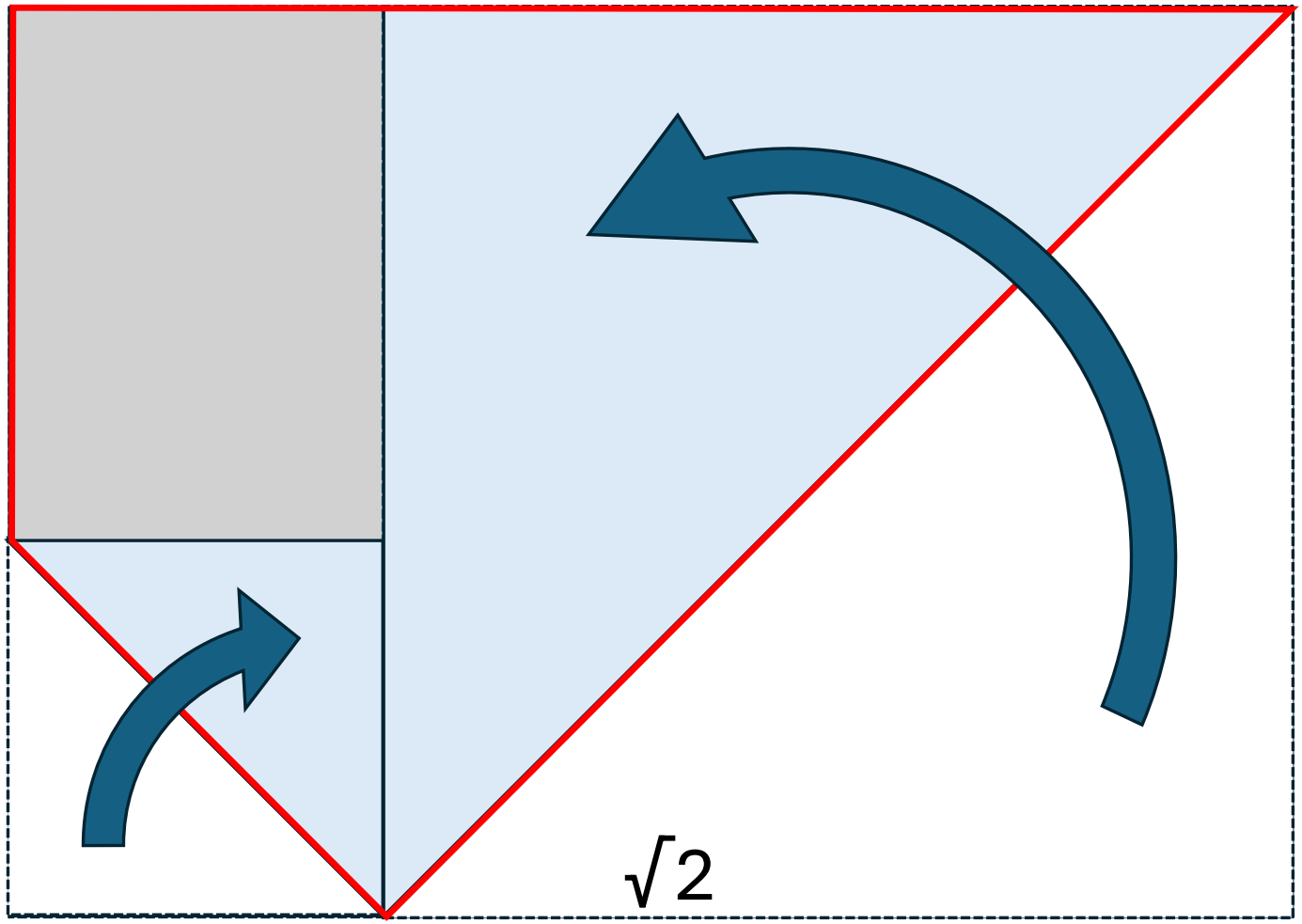
Paper mass

- Masses are given as gsm (grams per square metre): g m^{-2}
- A common mass for copy paper is 80 gsm.
  - That's 80 g for a sheet of A0.
  - So a sheet of A4 has a mass of $80/2^4$ = 5 g.
  - That's the same as a 20p piece, or a US nickel.

I brought scales and can prove it

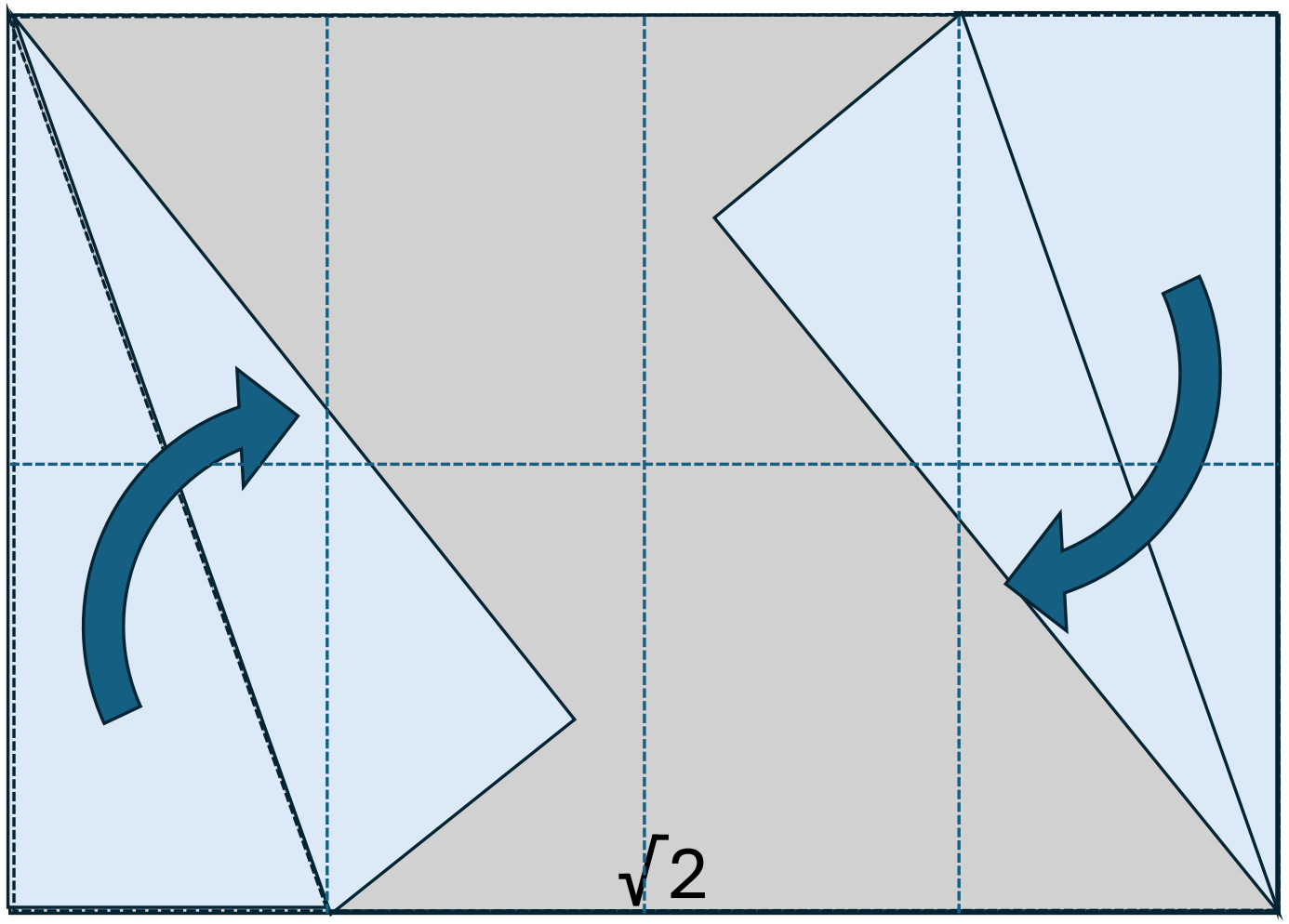
– Ask me later

Exploiting the √2 aspect ratio

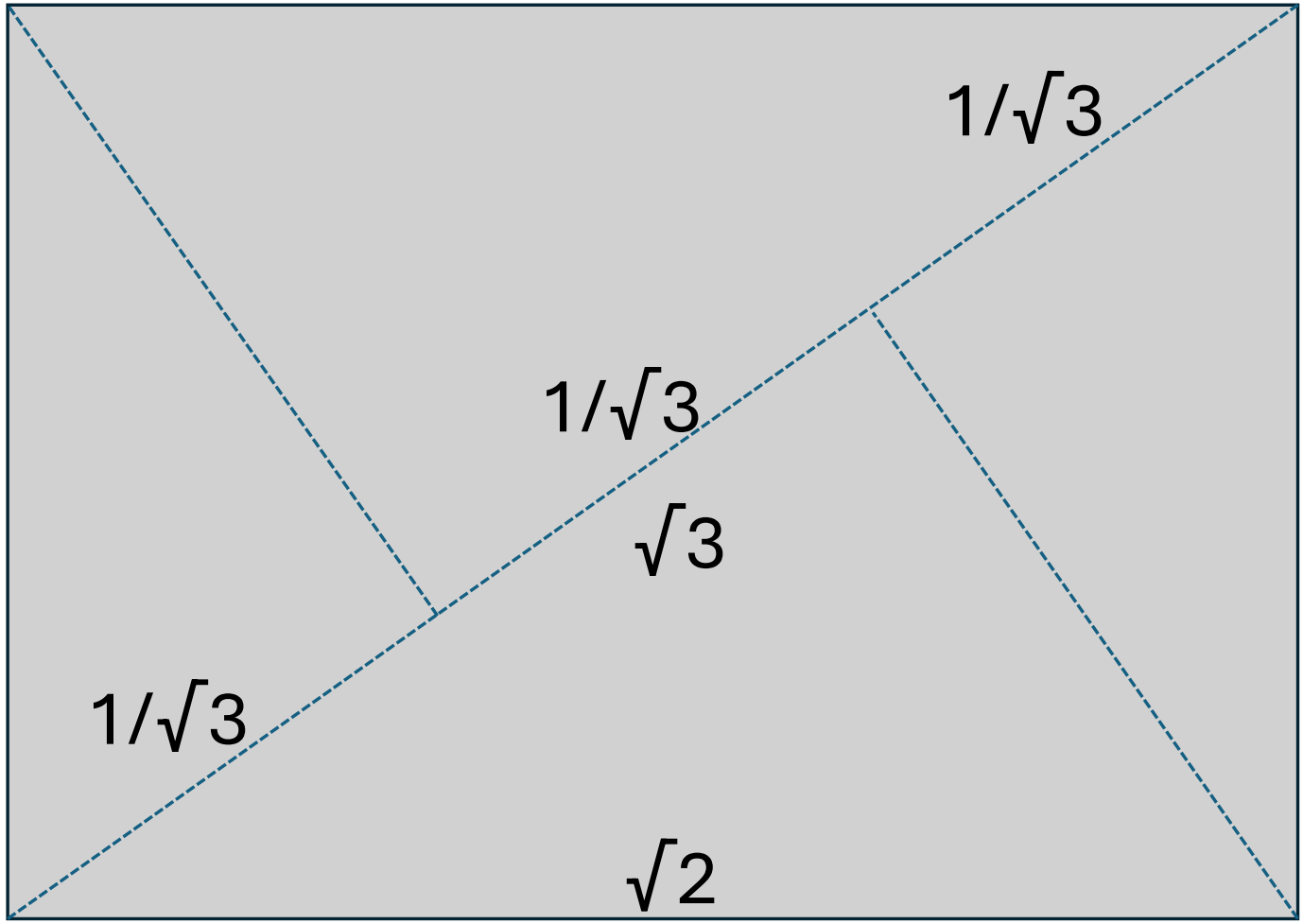


- Is this a kite?
- What is its perimeter?

Making a rhombus



Trisecting the diagonal

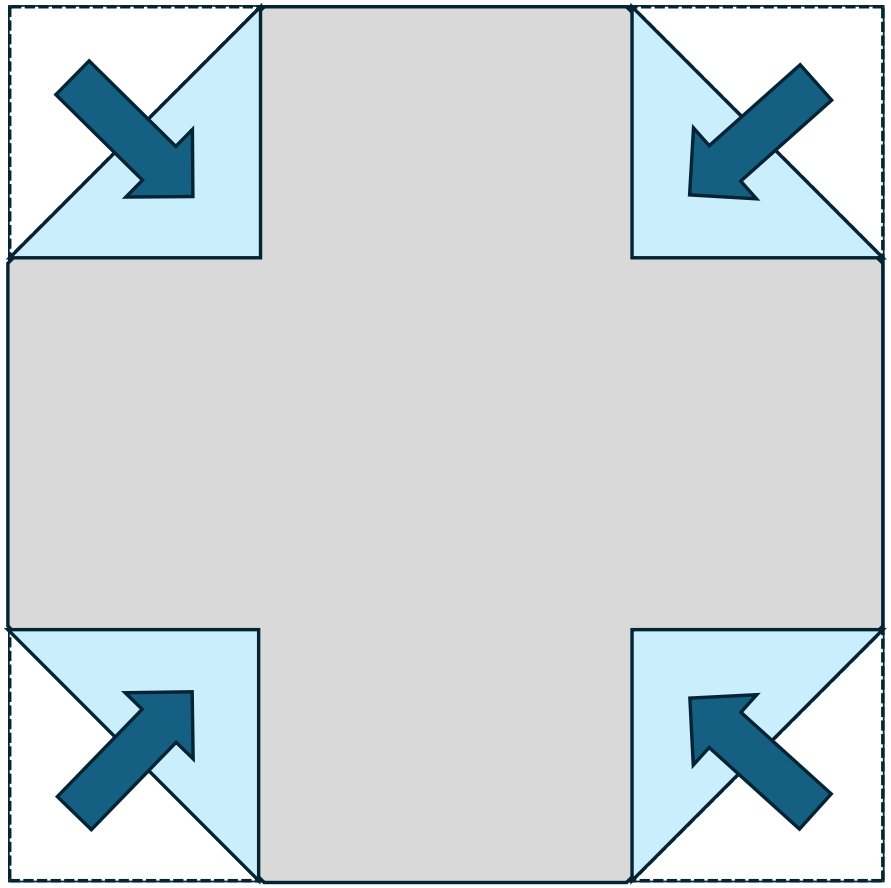


- Diagonal = √3
  - Pythagoras
- Dropping perpendiculars (or folding in) trisects that.
  - Similar triangles

Square paper – widely used for origami

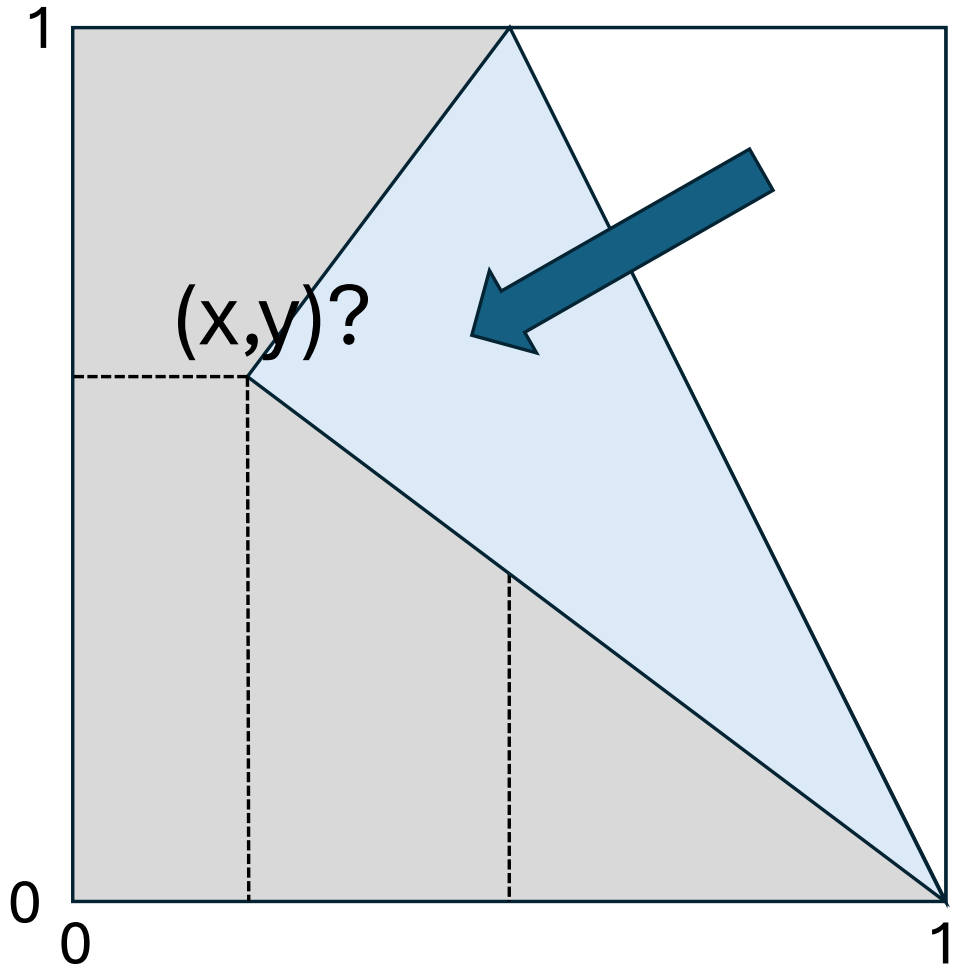
Square paper

How to fold it accurately into a regular octagon?



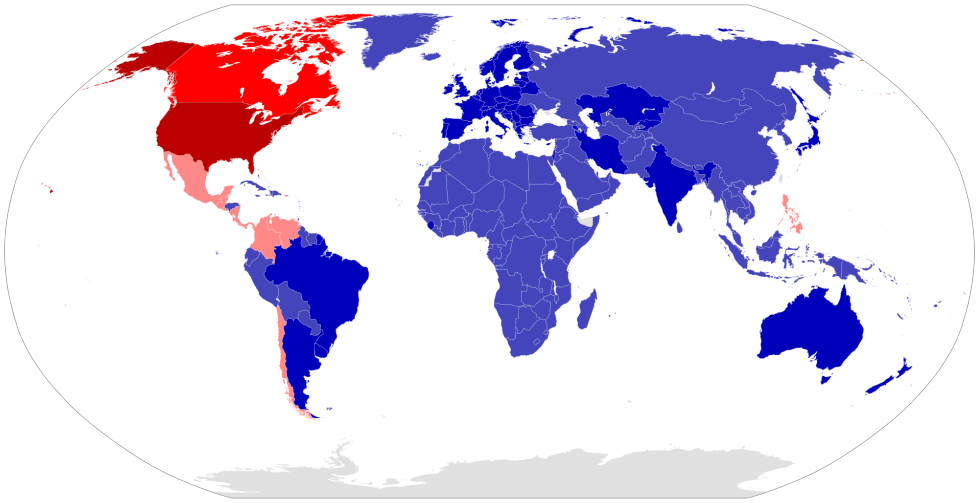
- Ask me later

Square paper



- What are the coordinates of this point?
- Ask me later – there's more…

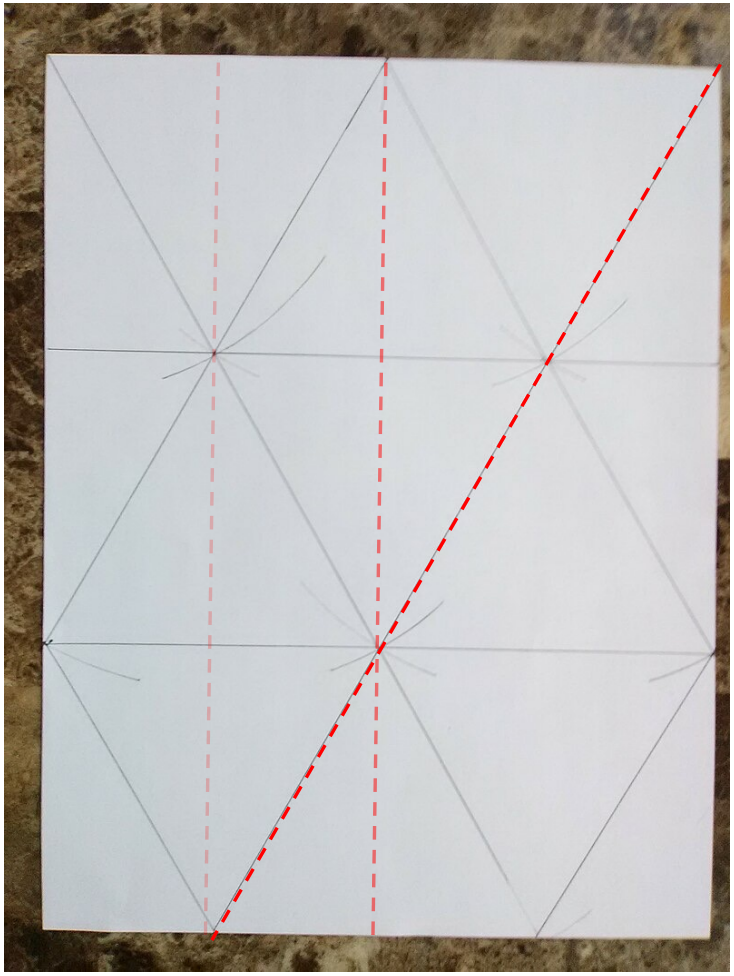
US sizes



US size	Width (inches)	Length (inches)
Letter	8 ½	11
Foolscap	8 ½	13
Foolscap folio	8 ½	13 ½
Legal	8 ½	14

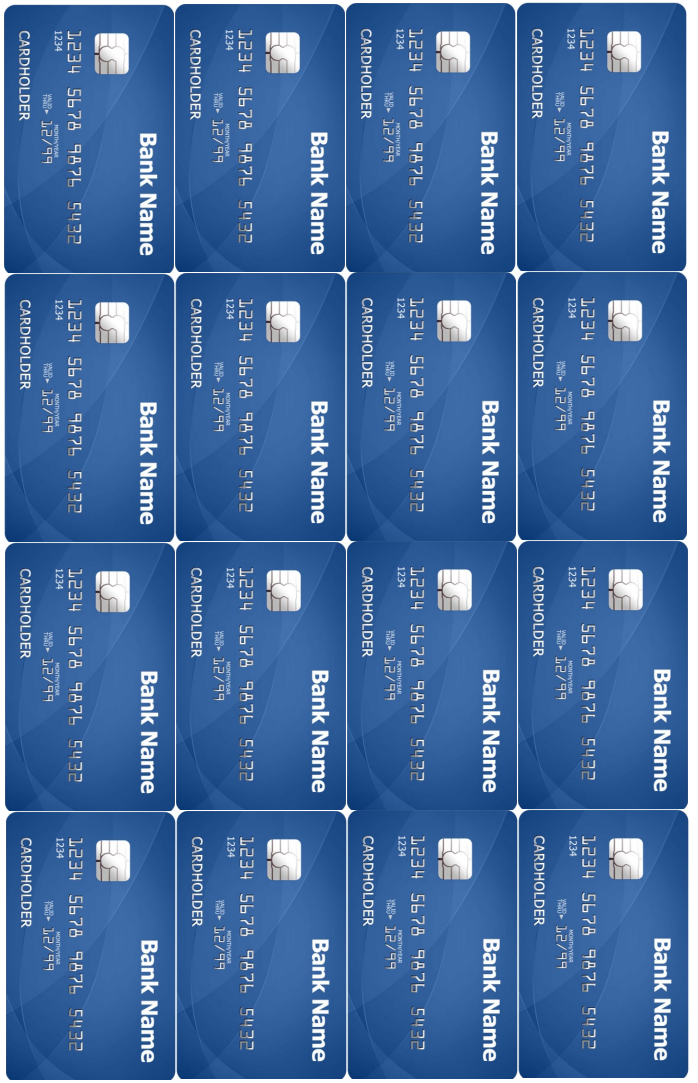
US Letter paper: 8 ½ ″ × 11 ″

- Aspect ratio = $11 / 8.5 \approx \frac{3}{4}\sqrt{3}$
  - To within 0.4%
- We can triangulate thus…
- What can it be folded into?



Credit cards

- Standardised
  - To work in machines worldwide: ISO/IEC 7810 ID-1
- 85.60 × 53.98 mm (based on 3 ⅜ " × 2 ⅛ ").
- ¼ of the dimensions of Foolscap folio (8.5 " × 13.5 ")



Foolscap folio

- 8.5 " × 13.5 "
- Converting to metric, that’s 215.9 × 342.9 mm.

To within 0.1mm accuracy, that’s $6^3 \times 7^3$ mm!