Dual numbers

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Arm®v8-M Architecture Reference Manual

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The elements of the vector registers are handled in pairs. In the base variant, corresponding elements from the two source registers are multiplied together, whereas the exchange variant swaps the values in each pair of values read from the first source register, before multiplying them with the values from the second source register. The results of the pairs of multiply operations are combined by adding them together. At the end of each beat these results are accumulated and the lower 32 bits written back to the general-purpose destination register. The initial value of the general-purpose destination register can optionally be added to the result.

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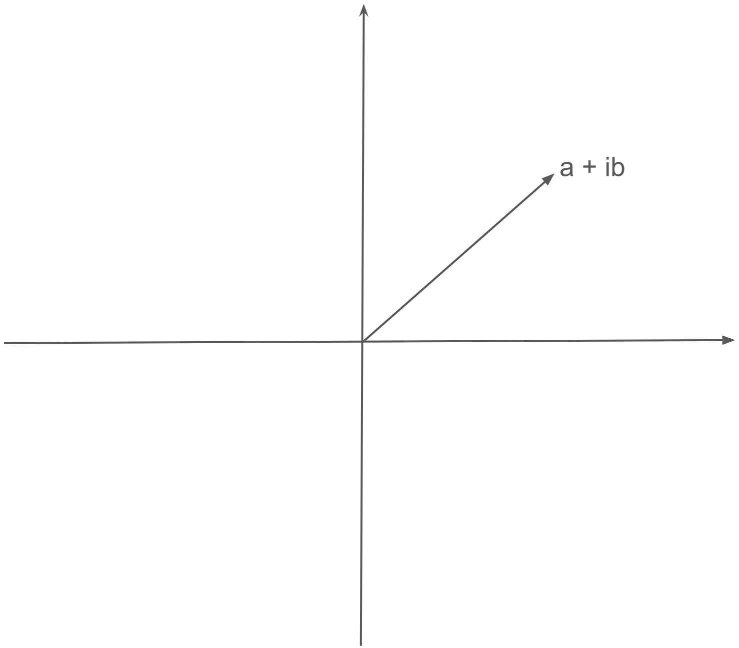
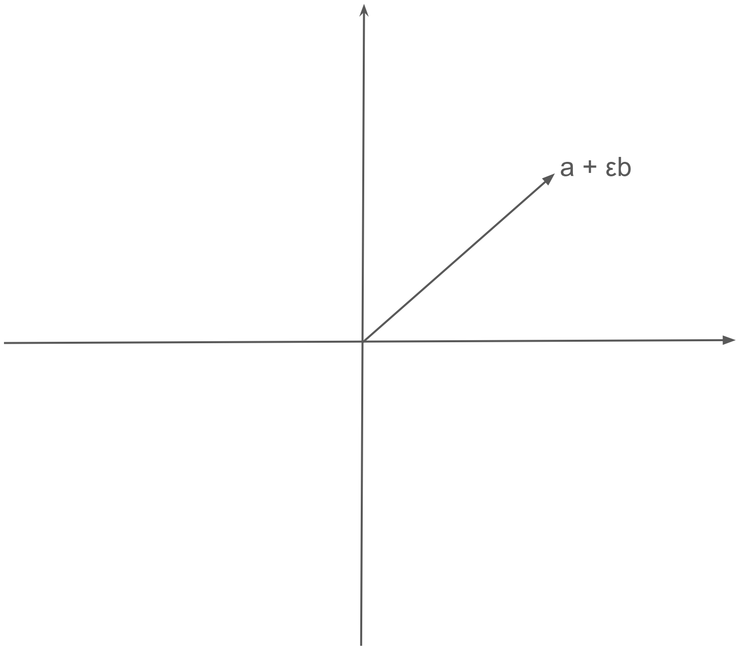
Complex numbers



- 2d vectors
- Write “a + ib” for (a, b)
- “Real” and “imaginary” parts
- $(a + ib) + (c + id) = (a + c) + i(b + d)$
- $i^2 = -1$
- $(a + ib)(c + id) = (ac - bd) + i(bc + ad)$

Dual numbers



- 2d vectors
- Write “$a + \varepsilon b$” for (a, b)
- “Body” and “soul”
- $(a + \varepsilon b) + (c + \varepsilon d) = (a + c) + \varepsilon(b + d)$
- $\varepsilon^2 = 0$
- $(a + \varepsilon b)\ (c + \varepsilon d) = (ac \cancel{+ bd}) + \varepsilon(bc + ad)$

You have seen these formulae before!

$(a + \varepsilon b) + (c + \varepsilon d) = (a + c) + \varepsilon(b + d)$

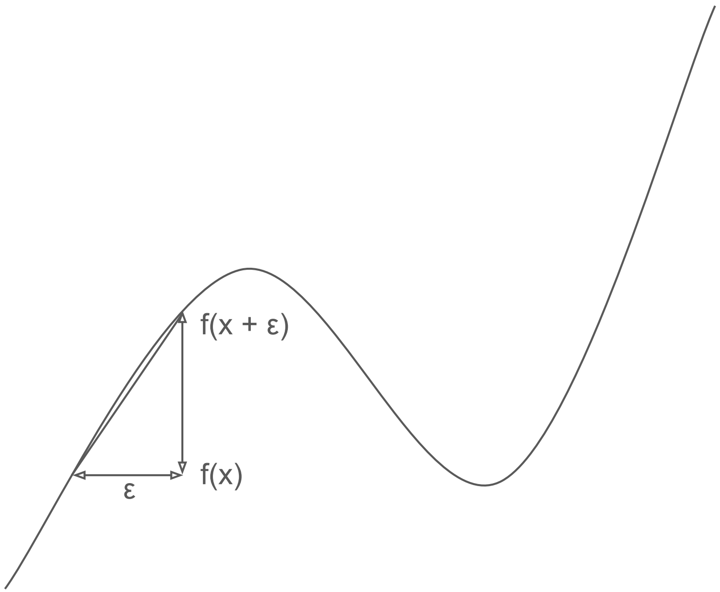
$(f(x) + g(x))' = f'(x) + g'(x)$

$(a + \varepsilon b)(c + \varepsilon d) = ac + \varepsilon(bc + ad)$

$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

Automatic differentiation

$f(x + \varepsilon) = f(x) + \varepsilon f'(x)$



Does it work?

Let $f(x) = x$

$f(x + \varepsilon) = x + \varepsilon$

Does it work?

Let $f(x) = x^2$

$f(x + \varepsilon) = (x + \varepsilon)^2$

$= x^2 + 2x\varepsilon + \varepsilon^2$

$= x^2 + 2x\varepsilon$

Does it work?

Let $f(x) = x^3$

$f(x + \varepsilon) = (x + \varepsilon)^3$

$= x^3 + 3x^2\varepsilon + 3x\varepsilon^2 + \varepsilon^3$

$= x^3 + 3x^2\varepsilon$

Does it work?

Let $f(x) = x^n$

$f(x + \varepsilon) = (x + \varepsilon)^n$

$= x^n + nx^{n-1}\varepsilon + \dots$

$= x^n + nx^{n-1}\varepsilon$

Yes!

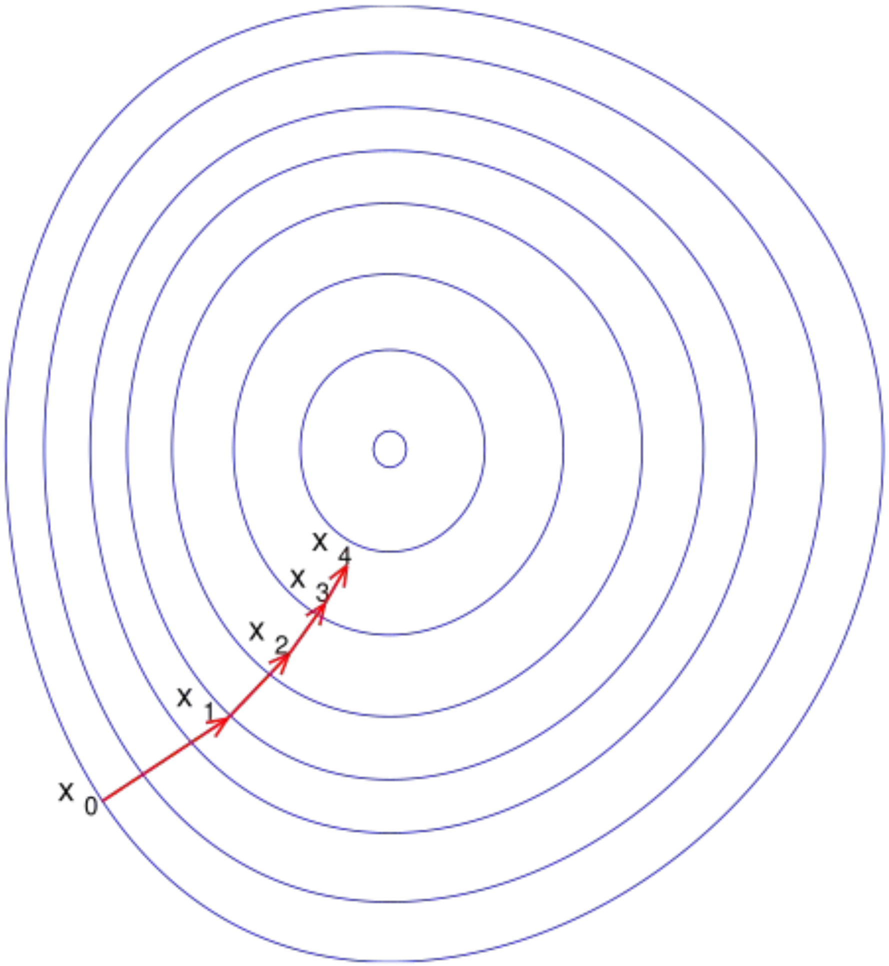
- Powers of n work
- Sums and products work
- So polynomial functions work
- So approximations to power series work
- But in fact, *arbitrary code* works
- We can extend this to higher or partial derivatives

Why do we care?

Physics simulations!

Computer graphics!

Gradient descent!



Historical note

- I first learned about these from Dan Piponi
- He used them in 2005 to do special effects for the *Matrix* sequels
- At the time they weren't widely-known in the graphics community
- But apparently Yoshua Bengio's deep-learning group already knew about them
    - "Source: rooftop beers" – Paul Khuong