MathsJam 2020: Box of Teasers

Here are some teasers, of varying challenge, to ponder and discuss over the MathsJam weekend.

Please email Yuen at twopointsevenoneeight@outlook.com for any corrections, comments or further puzzle suggestions.

I believe these teasers to be widespread, so for most of them I have not attributed any source.

1

You have 8 batteries; 4 of them work and 4 of them are flat. You also have a torch which will light up if two working batteries are placed into it.

How can you guarantee that the torch will light up in 7 (or fewer) attempts at putting batteries into the torch?

2

You have two containers. One can hold 7 litres of water and the other can hold 11 litres. You also have access to a sink with a tap. You can do any of these three things as many times as you like:

1. Fill one of the containers to the top with water.
2. Completely empty one of the containers into the sink.
3. Pour the contents of one container into the other until the second container is full (or until the first container is empty).

Given these conditions, is it possible to measure out exactly 5 litres of water?

3

Suppose that n people go to a meeting, where n>2, and that each person shakes hands with some, all or none of the other people. Show that there must be two people who have each shaken hands with the same number of people.

4

Two prisoners are locked up in a tower. They are held in separate cells and cannot see or otherwise communicate with each other.

The tower is situated on a field divided into two halves. One prisoner can see the south field only and the other prisoner can see the north field only.

There are some sheep in each field and the sheep cannot cross between the halves. The prisoners are told at the start that there are either 10 sheep or 13 sheep in total across both halves.

At 6pm exactly each day, if a prisoner is sure of the total number of sheep in both halves, he rings a bell in his cell to alert the warden, and goes free if his answer is correct. The prisoners are not permitted to ring the bell at any other time, do not guess and only inform the warden if they are 100% sure. The bell is loud enough for the other prisoner to hear.

At the end of the fifth day, both prisoners ring the bell at 6pm and are released. What is the number of sheep in each half?

5

A wooden box has the 5 letters {B A N D E} inscribed on it, and under each letter is an LED. On the back there are inscribed the 5 words {BAN DEAN BEN AND BEA}, and under each word is a button.

Pressing a button will affect all the LEDs of the letters in the word above the button: any LEDs that were off are switched on, and vice versa.

If all the LEDs are initially off, which buttons should be pressed to turn them all on?

6

Abi and Bess played 10 rounds of rock-paper-scissors. How many rounds did Abi win, given the facts below?

- Abi played rock 3 times, scissors once and paper 6 times
- Bess played rock 2 times, scissors 4 times and paper 4 times
- No ties occurred and the game order is unknown

7

60 people queue outside a 60-seater cinema. There are ten rows of six seats, with two aisles on either side, and seats must be accessed from either the left side or the right side of a row.

Each person receives a different, randomly chosen seat ticket. Everyone then take their seats, but each only enters the cinema once those queueing ahead of them have sat down already.

One row of seats has seats 31 to 33 closest to the left aisle and seats 34 to 36 closest to the right aisle.
Suppose that this row of seats is filled with people choosing their aisles for entering optimally (meaning they tried not to step over anybody else's feet) and that these six succeeded in not stepping over feet.
What was the probability of this occurring, and can you explain how to work out this probability?

8

A nurse is tasked with measuring the individual weights of quintuplets. However, the naughty children will only stand on the weighing scales in pairs at a time.

The nurse records 10 measurements, in kg, of 50, 52, 53, 54, 55, 56, 57, 58, 60 and 61.

What are the weights of each child?

9

Using any number of the symbols +, -, x, /, (, ), and the digits 1, 3, 4 and 6 once, create an expression equalling 24.

10

The statements A-F refer to themselves.
Only one of the six statements A-F is true.
Which one is it?

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A. All of the below
B. None of the below
C. One of the above
D. All of the above
E. None of the above
F. None of the above

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11

Alice and Bob play a game in which the ‘key number’ starts as 2019.

A turn consists of replacing the key number with a non-negative number that is an integer power of 2 fewer than the current key number.

Alice and Bob take turns, one after another, until someone leaves behind a key number of 0. This person will win the game.

Alice gets to decide whether to go first or second. What is her winning strategy?

12

Form a list of words, starting with ‘startling’, such that each word (except the first) has all but one of the letters of the previous word.

13

I have picked one colour (from black and white) and one shape (from round and square). If a symbol possesses exactly one of the two properties I have picked, then let us call it a ‘thog’.

If I reveal that the black circle is a ‘thog’, then what can be said about the other three symbols?

1. Definitely a thog
2. Undecidable
3. Definitely not a thog

14

A doctor and a surgeon each walk up an ascending escalator.

The doctor takes 20 steps and reaches the top in 60 seconds.

The surgeon takes 16 steps and reaches the top in 72 seconds.

A firefighter starts from the top of the escalator and takes 120 seconds to reach the bottom.

Explain how to work out the number of steps the firefighter took, and what was this number?

15

What is the quickest possible way of measuring 9 minutes, using only a 4-minute sandtimer and a 7-minute sandtimer?

A sandtimer can measure a fixed amount of time: simply turn the sandtimer upside-down and wait for all the sand grains to fall from the top half to the bottom half.

16

A) At each vertex of a triangle lies exactly one ant. Each of the three ants chooses at random one of the two edges it can walk down. Assuming they each walk at the same steady speed, what are the chances that at least one collision occurs?

B) At each vertex of a hexagon lies exactly one ant. Each of the six ants chooses at random one of the two edges it can walk down. Assuming they each walk at the same steady speed, what are the chances that at least one collision occurs?

17

A game show host presents three doors numbered 1, 2 and 3 to you. She explains that behind one door is a car and behind each of the others is a goat, and that she knows where the car is.

You pick door 1 to open but the host trips, and while falling she accidentally knocks door 3 open, revealing a goat.

You now get the option to switch to door 2. Explain whether switching doors improves your chances of winning a car.

18

Diagram A shows 7 dots in an 'H' shape. Diagram B shows the same dots with 5 lines running through them. Each line goes through at least 3 dots.

Can you add 2 new dots to diagram A so that there exist 10 distinct lines running through them? Each line must go through at least 3 dots.

(Up to symmetry there are at least 2 different ways to do this.)

Diagram A

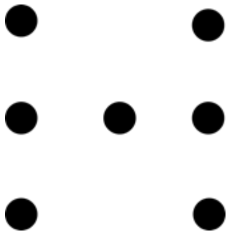
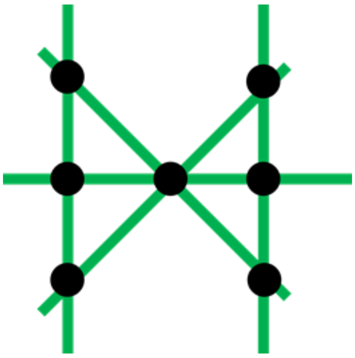


Diagram B



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As ever – if there are any comments, corrections or even puzzle suggestions, please contact Yuen on twopointsevenoneeight@outlook.com ☺